Journal of Engineering Physics and Thermophysics, Vol. 82, No. 1, 2009

MISCELLANEOUS

MODEL OF THE INTERACTION OF ELECTROMAGNETIC AND THERMAL FIELDS IN DELAY MEDIA

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UDC 517.958;517.87

We have solved the problem on the propagation of a high-frequency electromagnetic field in a half-space filled with a medium with delayed electric and magnetic polarizations and conduction currents under the action on the half-space of an arbitrary combination of plane fields, as well as of surface currents and charges distributed over the half-space surface. A model of the heating of the half-space medium as a result of the electromagnetic-to-thermal and vice versa energy conversion has been considered. A numerical study of the heating depending on the delay time has been made.

Keywords: *Maxwell equations, plane electromagnetic waves, delay medium, boundary-value problem, heat conduction equation, energy dissipation.*

Introduction. In creating engineering units, induction heating of parts is widely used, which makes it possible to change the properties of the material and the surface layer of a part to the required quality [1]. For simple materials, the theory of interaction of electromagnetic and thermal fields has been well developed. In the last few years, much consideration has been given to the development of mathematical models of the interaction in composite and other complex materials [2]. In particular, in [3] a model of the electromagnetic field in materials with polarization delay with respect to the external electromagnetic field and with magnetic relaxation processes has been constructed.

Below we develop a model of the dissipation of electromagnetic energy of a flat monochromatic wave in materials with a simultaneous delay of electric and magnetic polarizations, as well as of conduction currents in view of the equations of [4, p. 20] and surface currents and charges. In so doing, we assume that with increasing temperature the electrophysical properties of the medium remain unchanged. It is also assumed that at $t < 0$ the medium is not heated and the switching process of the electromagnetic field is not taken into account.

Let us formulate the mathematical model in the form of a boundary-value problem describing the interaction of the plane electromagnetic wave with the half-space of the delay medium.

Statement of the Problem. Consider a plane interface Γ ($z = 0$) between two homogeneous media filling the half-spaces D_1 ($z < 0$) and D_2 ($z > 0$) (Fig. 1). The medium D_1 is defined by electrical ε_1 and magnetic permeability μ_1, and D_2 is a delay medium. In the region of D_1 the primary plane monochromatic electromagnetic field with complex amplitudes $\mathbf{E}_0$, $\mathbf{H}_0$ oscillating with circular frequency ω propagates. Denote by $\mathbf{E}_1'$, $\mathbf{H}_1'$ the complex amplitudes of the reflected field in the region of D_1, and by $\mathbf{E}_2$, $\mathbf{H}_2$ the field in the region of D_2; $\mathbf{E}_1 = \mathbf{E}_0 + \mathbf{E}_1'$, $\mathbf{H}_1 = \mathbf{H}_0 + \mathbf{H}_1'$ is the total field in D_1.

As is known, the field in the region of D_1 obeys the Maxwell equations for complex amplitudes

$$\text{rot}\,\mathbf{H}_1 = -i\omega\varepsilon_1\mathbf{E}_1\,, \quad \text{rot}\,\mathbf{E}_1 = i\omega\mu_1\mathbf{H}_1\,. \qquad (1)$$

For the real field $\mathbf{E}$, $\mathbf{H}$ in the region of D_2 (1) the Maxwell equation holds for the delay medium

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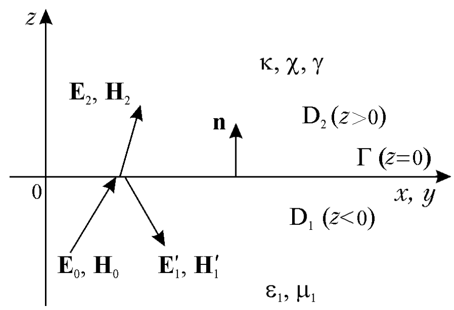

Fig. 1. Interface between two homogeneous media.

$$\text{rot}\,\mathbf{H} = \frac{\partial \mathbf{D}}{\partial t} + \mathbf{J}, \quad \text{rot}\,\mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}, \tag{2}$$

where $\mathbf{D} = \varepsilon_0\mathbf{E}(t) + \mathbf{P}(t)$; $\mathbf{B} = \mu_0\mathbf{H}(t) + \mathbf{m}(t)$; $\mathbf{J} = \gamma\mathbf{E}(t - t_0)$; $\mathbf{P}(t) = \varepsilon_0\kappa\mathbf{E}(t - t_1)$; $\mathbf{m}(t) = \mu_0\chi\mathbf{H}(t - t_2)$.

Let us write Eqs. (2), having transformed them to the equations for complex amplitudes of the monochromatic field $\mathbf{E}_2$, $\mathbf{H}_2$, in the form [4]

$$\text{rot}\,\mathbf{H}_2 = -i\omega\varepsilon_2\,\mathbf{E}_2, \quad \text{rot}\,\mathbf{E}_2 = i\omega\mu_2\,\mathbf{H}_2, \tag{3}$$

where

$$\varepsilon_2 = \varepsilon_0\left(1 + \kappa\exp(i\omega t_1)\right) + i\frac{\gamma}{\omega}\exp(i\omega t_0); \quad \mu_2 = \mu_0\left(1 + \chi\exp(i\omega t_2)\right);$$

$$\mathbf{E}(t) = \text{Re}\left(\mathbf{E}_2\exp(-i\omega t)\right); \quad \mathbf{H}(t) = \text{Re}\left(\mathbf{H}_2\exp(-i\omega t)\right). \tag{4}$$

Suppose that on the surface of Γ surface charges with a surface complex density σ and surface currents with a surface complex density of currents **j** are induced. In this connection, at the interface Γ the boundary conditions

$$\left[\mathbf{n}, [(\mathbf{E}_2 - \mathbf{E}_1), \mathbf{n}]\right]\Big|_{z=0} = 0, \quad \left[\mathbf{n}, [(\mathbf{H}_2 - \mathbf{H}_1), \mathbf{n}]\right]\Big|_{z=0} = [\mathbf{j}, \mathbf{n}]; \tag{5}$$

$$(\mu_2\mathbf{H}_2, \mathbf{n})|_{z=0} = (\mu_1\mathbf{H}_1, \mathbf{n})|_{z=0}, \quad (\varepsilon_2\mathbf{E}_2, \mathbf{n})|_{z=0} = (\varepsilon_1\mathbf{E}_1, \mathbf{n})|_{z=0} + \sigma; \tag{6}$$

$$i\omega\sigma = \text{div}\,\mathbf{j}, \tag{7}$$

where the vector **j** is tangent to the Γ plane and $\mathbf{n} = \mathbf{e}_z$ is the normal to it, are fulfilled.

The electromagnetic fields defining the solution of problem (1), (3), (5)–(7) should satisfy the infinity conditions, reducing to the fact that the field $\mathbf{E}_2$, $\mathbf{H}_2$ should attenuate in the positive direction of the *Oz* axis and the energy of the reflected field $\mathbf{E}_1'$, $\mathbf{H}_1'$ should propagate in the negative direction.

Analytic Representation of the Solution of the Problem. Let us choose for the primary plane field a linear combination of TH-polarized (vector **H** is orthogonal to the *Oz* axis, vector **E** is not orthogonal to the *Oz* axis in the general case) and TE-polarized (vector **E** is orthogonal to the *Oz* axis, vector **H** is not orthogonal to the *Oz* axis in the general case) fields incident at an angle with the plane of the interface Γ, and express them in terms of the basic plane fields [5] in the medium D_1 in the form

$$\mathbf{E}_0 = A\mathbf{W}^{(-1)}(\mathbf{r}; \alpha_1, \alpha_2; k_1) + B\mathbf{W}^{(-2)}(\mathbf{r}; \alpha_1, \alpha_2; k_1),$$

$$\mathbf{H}_0 = \frac{k_1}{i\omega\mu_1}\left[A\mathbf{W}^{(-2)}(\mathbf{r}; \alpha_1, \alpha_2; k_1) + B\mathbf{W}^{(-1)}(\mathbf{r}; \alpha_1, \alpha_2; k_1)\right], \tag{8}$$

where by definition

$$\mathbf{W}^{(\mp 1)}(\mathbf{r}; \alpha_1, \alpha_2; k) = \frac{i}{\lambda}\mathbf{e}_1 \exp(i\alpha_1 x + i\alpha_2 y \mp vz);$$

$$\mathbf{W}^{(\mp 2)}(\mathbf{r}; \alpha_1, \alpha_2; k) = \frac{1}{k}\left(\mp\frac{iv}{\lambda}\mathbf{e}_2 + \lambda\mathbf{e}_z\right)\exp(i\alpha_1 x + i\alpha_2 y \mp vz); \tag{9}$$

$\mathbf{r} = (x, y, z)$; $\mathbf{e}_1 = \alpha_2\mathbf{e}_x - \alpha_1\mathbf{e}_y$; $\mathbf{e}_2 = \alpha_1\mathbf{e}_x + \alpha_2\mathbf{e}_y$; $\lambda = \sqrt{\alpha_1^2 + \alpha_2^2}$; $v = \sqrt{\lambda^2 - k^2}$; $k = \omega\sqrt{\varepsilon\mu}$; $0 \le \arg\lambda,\ k < \pi$; $-\pi/2 \le \arg v < \pi/2$; the complex values of α_1, α_2 determine the inclination of the primary field with respect to the Γ plane.

The reflected and transmitted through the Γ plane fields have the same structure [6] as does the primary field; therefore, we also represent them in terms of the basic plane fields (9) in the form of linear combinations satisfying the Maxwell equations (1) and (3). As a result, we have representations

$$\mathbf{E}_1' = x_1\mathbf{W}^{(+1)}(\mathbf{r}; \alpha_1, \alpha_2; k_1) + y_1\mathbf{W}^{(+2)}(\mathbf{r}; \alpha_1, \alpha_2; k_1),$$

$$\mathbf{H}_1' = \frac{k_1}{i\omega\mu_1}\left[x_1\mathbf{W}^{(+2)}(\mathbf{r}; \alpha_1, \alpha_2; k_1) + y_1\mathbf{W}^{(+1)}(\mathbf{r}; \alpha_1, \alpha_2; k_1)\right], \quad z < 0; \tag{10}$$

$$\mathbf{E}_2 = x_2\mathbf{W}^{(-1)}(\mathbf{r}; \alpha_1, \alpha_2; k_2) + y_2\mathbf{W}^{(-2)}(\mathbf{r}; \alpha_1, \alpha_2; k_2),$$

$$\mathbf{H}_2 = \frac{k_2}{i\omega\mu_2}\left[x_2\mathbf{W}^{(-2)}(\mathbf{r}; \alpha_1, \alpha_2; k_2) + y_2\mathbf{W}^{(-1)}(\mathbf{r}; \alpha_1, \alpha_2; k_2)\right], \quad z > 0, \tag{11}$$

where the constants x_j, y_j, $j = 1, 2$ are to be determined. Note that the structure of fields (10), (11) was chosen so that the infinity conditions were fulfilled automatically.

The primary field-induced charges and currents on the Γ plane have an exponential dependence along the axis x and y coordinates since fields (10), (11) defining the solution of the problem depend exponentially on the spatial coordinates. Therefore,

$$\sigma = \sigma_0\Phi, \quad \Phi = \exp(i\alpha_1 x + i\alpha_2 y). \tag{12}$$

Let us divide the current density vector into a potential and a solenoidal parts giving them in the form $\mathbf{j}$ = grad ψ + rot Ψ, where $\psi = j_p\Phi$, $\Psi = j_s\Phi\mathbf{e}_z$; j_p, j_s — const. As a result,

$$\mathbf{j} = i\,(j_s\mathbf{e}_1 + j_p\mathbf{e}_2)\,\Phi, \quad [\mathbf{j}, \mathbf{e}_z] = i\,(j_p\mathbf{e}_1 - j_s\mathbf{e}_2)\,\Phi. \tag{13}$$

Substitute (12), (13) into condition (7) and find the dependence $j_p = \omega\sigma_0/(i\lambda^2)$.

Calculation of the Field Amplitudes. Let us satisfy the boundary conditions (5) having calculated preliminarily the tangential components of fields (8)–(11) on the $z = 0$ plane. We get

$$\mathbf{E}_{1\tau}' = \frac{i}{\lambda}\left(x_1\mathbf{e}_1 + \frac{v_1}{k_1}y_1\mathbf{e}_2\right)\Phi, \quad \mathbf{H}_{1\tau}' = \frac{k_1}{\omega\mu_1\lambda}\left(y_1\mathbf{e}_1 + \frac{v_1}{k_1}x_1\mathbf{e}_2\right)\Phi;$$

$$\mathbf{E}_{2\tau} = \frac{i}{\lambda}\left(x_2\mathbf{e}_1 - \frac{v_2}{k_2}y_2\mathbf{e}_2\right)\Phi, \quad \mathbf{H}_{2\tau} = \frac{k_2}{\omega\mu_2\lambda}\left(y_2\mathbf{e}_1 - \frac{v_2}{k_2}x_2\mathbf{e}_2\right)\Phi; \tag{14}$$

$$\mathbf{E}_{0\tau}=\frac{i}{\lambda}\left(A\mathbf{e}_1-\frac{v_1}{k_1}B\mathbf{e}_2\right)\Phi\,,\quad \mathbf{H}_{0\tau}=\frac{k_1}{\omega\mu_1\lambda}\left(B\mathbf{e}_1-\frac{v_1}{k_1}A\mathbf{e}_2\right)\Phi\,.$$

Substitute expressions (13), (14) into the boundary conditions (5) and equate the coefficients at linearly independent vectors $\mathbf{e}_1$, $\mathbf{e}_2$. We obtain systems of linear algebraic equations for determining the coefficients x_j, y_j:

$$x_2-x_1=A\,,\quad \frac{\mu_1 v_2}{\mu_2 v_1}x_2+x_1=A+\frac{i\omega\mu_1\lambda}{v_1}j_{\mathrm{s}}\,;\quad \frac{k_1 v_2}{k_2 v_1}y_2+y_1=B\,,\quad \frac{\mu_1 k_2}{\mu_2 k_1}y_2-y_1=B+\frac{i\omega\mu_1\lambda}{k_1}j_{\mathrm{p}}\,.$$

Solving these systems, we find the values of the coefficients for the sums of (10), (11):

$$x_2=\frac{(2A+g_1 j_{\mathrm{s}})\,\mu_2 v_1}{\mu_2 v_1+\mu_1 v_2}\,,\quad x_1=\frac{(\mu_2 v_1-\mu_1 v_2)\,A+\mu_2 v_1 g_1 j_{\mathrm{s}}}{\mu_2 v_1+\mu_1 v_2}\,;$$

$$y_2=\frac{(2B+f_1\sigma_0)\,k_1 k_2 v_1 \mu_2}{\mu_1 v_1 k_2^2+\mu_2 v_2 k_1^2}\,,\quad y_1=\frac{\left(\mu_1 v_1 k_2^2-\mu_2 v_2 k_1^2\right)B-f_1 k_1^2 v_2 \mu_2 \sigma_0}{\mu_1 v_1 k_2^2+\mu_2 v_2 k_1^2}\,, \tag{15}$$

where $f_1 = k_1/\lambda\varepsilon_1$; $g_1 = i\omega\mu_1\lambda/v_1$; $v_j = \sqrt{\lambda^2-k_j^2}$; $-\pi/2 \le \arg v_j < \pi/2$; $k_j = \omega\sqrt{\varepsilon_j\mu_j}$; $0 \le \arg k_j < \pi$, $j = 1, 2$.

Note that the boundary conditions (6) are fulfilled automatically since they are a consequence of relations (5), (7).

Formulas (15) specifying the solution of (10), (11) show that the interaction of the plane field with the plane interface between the media is determined by the primary field amplitudes A and B, the surface charges σ_0, and the amplitude of surface eddy currents $I_{\mathrm{s}} = i\lambda j_{\mathrm{s}}$, where λ is a real number. The complex values of A, B, σ_0, j_{s} are assumed to be given.

To calculate the real field in the D_2 region, formulas (4) are used.

Induction Heating of the Half-Space. Let us distinguish the real and the imaginary part of the dielectric constant and magnetic permeability $\varepsilon_2 = \varepsilon_{\mathrm{Re}} + i\varepsilon_{\mathrm{Im}}$, $\mu_2 = \mu_{\mathrm{Re}} + \mu_{\mathrm{Im}}$. Then the power density of the electromagnetic energy absorbed and converted to heat and vice versa in the D_2 half-space is defined in terms of field (4) by the formula

$$\Pi=\mathbf{E}\frac{\partial\mathbf{P}}{\partial t}+\mathbf{H}\frac{\partial\mathbf{m}}{\partial t}+\mathbf{EJ}=\frac{1}{2}(\varepsilon_{\mathrm{Re}}-\varepsilon_0)\frac{\partial\mathbf{E}^2}{\partial t}+\frac{1}{2}(\mu_{\mathrm{Re}}-\mu_0)\frac{\partial\mathbf{H}^2}{\partial t}+\omega\left(\varepsilon_{\mathrm{Im}}\mathbf{E}^2+\mu_{\mathrm{Im}}\mathbf{H}^2\right),\quad z>0\,. \tag{16}$$

The process of heat release is described by the heat conduction equation

$$C\rho\frac{\partial u}{\partial t}-K_{\mathrm{h}}\Delta u=\Pi\,, \tag{17}$$

where $u = u(\mathrm{M}, t)$ is the medium temperature at point $\mathrm{M} \in D_2$; Π is the density of heat sources.

For high frequencies, the thermal process has no time to keep up with a change in the electromagnetic field. Therefore, let us simplify the model by averaging function (16) over the field period $T = 2\pi/\omega$. We obtain [4, 7]

$$F=\frac{1}{T}\int_0^T\Pi(t)\,dt=\frac{\omega}{2}\left(\varepsilon_{\mathrm{Im}}|\mathbf{E}_2|^2+\mu_{\mathrm{Im}}|\mathbf{H}_2|^2\right).$$

In view of (11), for the real values of the field α_1 and α_2 we define

$$F=F(z)=f\exp(-gz)\,,\quad g=2\operatorname{Re}v_2\,, \tag{18}$$

where

$$f = \frac{\omega}{2}\left\{\frac{1}{\lambda^2}\left[\varepsilon_{\text{Im}}\left(\left|\alpha_2 x_2 - \alpha_1 y_2 \frac{v_2}{k_2}\right|^2 + \left|\alpha_1 x_2 + \alpha_2 y_2 \frac{v_2}{k_2}\right|^2\right)\right.\right.$$

$$\left.\left. + \frac{\mu_{\text{Im}}}{\omega^2 |\mu_2|^2}\left(|k_2\alpha_2 y_2 - \alpha_1 v_2 x_2|^2 + |\alpha_1 k_2 y_2 + \alpha_2 v_2 x_2|^2\right)\right] + \lambda^2\left(\varepsilon_{\text{Im}}\frac{|y_2|^2}{|k_2|^2} + \frac{\mu_{\text{Im}}|x_2|^2}{\omega^2|\mu_2|^2}\right)\right\}.$$

Considering the heat conduction equation (17) for the averaged heat sources (18), we formulate the following initial-boundary-value problem:

$$\frac{\partial u}{\partial t} - a^2 \frac{\partial^2 u}{\partial z^2} = \frac{F(z)}{\rho C}, \quad 0 < z < \infty, \quad 0 < t < \infty,$$
$$u|_{t=0} = 0, \quad z \geq 0; \quad K_{\text{h}} \left.\frac{\partial u}{\partial z}\right|_{z=0} = p, \quad t \geq 0, \tag{19}$$

where $a = \sqrt{K_{\text{h}}/\rho C}$; p is a given heat flow through the Γ plane (p = const).

Let us give the solution problem (19) analytically:

$$u(z, t) = -G \exp(-gz) + \frac{G}{2}(Q(z, t) + Q(-z, t)) + \frac{P}{K_{\text{h}}}\left(z \operatorname{erfc}\left(\frac{z}{2a\sqrt{t}}\right) - 2a\sqrt{\frac{t}{\pi}} \exp\left(-\frac{z^2}{4a^2 t}\right)\right), \tag{20}$$

where $Q(z, t) = \exp(gz + a^2 g^2 t \operatorname{erfc}\left(ag\sqrt{t} + \frac{z}{2a\sqrt{t}}\right)$; $G = \frac{f}{\rho a^2 g^2 C}$; $P = p - K_{\text{h}} G g$; erfc $(z) = 1 -$ erf (z), erf (z) is the probability integral [8, p. 120].

Calculation of the Averaged Temperature *u*. Let us define the parameters of the D_1 medium assuming $\varepsilon_1 = \varepsilon_0$, $\mu_1 = \mu_0$, $\gamma_1 = 0$, $k_1 = \omega/c$. For the primary field (8) $A = 5\cdot 10^3$ V/m, $B = 0$ V/m, $\omega = 2\pi\cdot 10^5$ sec^{-1}, $\alpha_1 = k_1 \cos\varphi_0 \sin\theta_0$, $\alpha_2 = k_1 \sin\varphi_0 \sin\theta_0$, $\varphi_0 = \pi/4$, $\theta_0 = \pi/5$; ($\cos\varphi_0 \sin\theta_0$, $\sin\varphi_0 \sin\theta_0$, $\cos\theta_0$) is the direction of propagation of the primary field. As is seen, the vector $\mathbf{E}_0$ is parallel to the Γ plane. For the medium in the D_2 half-space, consider different variants of the delay time for conduction currents, as well as for the electric and magnetic polarizations. Let us investigate graphically the heating of the D_2 half-space exposed to the primary field in view of formula (20) (see Fig. 2).

The right-hand side of Eq. (19) models the density of averaged heat sources created by the monochromatic electromagnetic field. The sign of the right-hand side is determined by the sign of the factor f in formula (18). In the case of $f > 0$ Eq. (19) describes the heating of the medium in the D_2 half-space, and in the case of $f < 0$ it describes the cooling of the medium. As is seen, the sign of the real factor f is determined by the sign of the imaginary parts

$$\varepsilon_{\text{Im}} = \varepsilon_0 \kappa \sin(2\pi\xi_1) + \frac{\gamma}{\omega}\cos(2\pi\xi_0), \quad \mu_{\text{Im}} = \mu_0 \chi \sin(2\pi\xi_2) \tag{21}$$

of the complex characteristics of the medium ε_2, μ_2. In so doing, delay times comparable to the field period T $t_m = \xi_m T$ $(0 \leq \xi_m \leq 1)$, are considered. For passive media at $\varepsilon_{\text{Im}} > 0$, $\mu_{\text{Im}} > 0$ [7, p. 34 (9.11)] $f > 0$ follows, i.e., Eq. (19) models the electromagnetic-to-thermal energy conversion. Analysis of formulas (21) shows that always $\varepsilon_{\text{Im}} > 0$, $\mu_{\text{Im}} > 0$ at $0 < t_m < T/4$.

In the case of active media, at $\varepsilon_{\text{Im}} < 0$, $\mu_{\text{Im}} < 0$ [7, p. 34] we obtain $f < 0$ and thermal energy is converted to electromagnetic energy. At $\varepsilon_{\text{Im}} > 0$, $\mu_{\text{Im}} < 0$ ($\varepsilon_{\text{Im}} < 0$, $\mu_{\text{Im}} > 0$) the heating and cooling depend on the system parameters.

Figures 2 and 3 show the dependences of the temperature u of passive media on $l = z/h$ ($h = (\text{Re}\, v_2)^{-1}$ is the skin-layer depth in D_2; $l = 1, 2, ..., \infty$) at the following parameters: $\omega = 2\pi\cdot 10^5$ 1/sec, $\kappa = 50$, $\chi = 10^2$, $\gamma = 10/(\Omega\cdot\text{m})^{-1}$, $C = 5\cdot 10^{-2}$ J/(K·kg), $\rho = 8\cdot 10^3$ kg/m^3, $K_{\text{h}} = 50$ kg·m/(sec^3·K), $j_s = 0$, $u_0 = 273$ K.

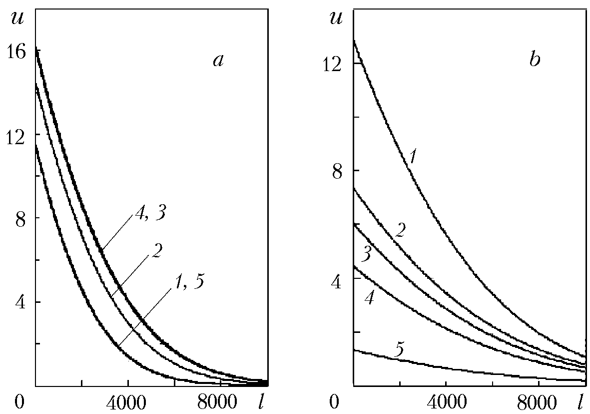

Fig. 2. Temperature distribution in the half-space depth in the absence of heat flow and at $\sigma_0 = 0$: a) $t = 10^3$ sec, $\xi_0 = 0$, $\xi_1 = 0$ (1) $\xi_2 = 0$, $h = 5{\cdot}10^{-3}$ m; 2) 1/5 and $3.6{\cdot}10^{-3}$; 3) 1/4 and $3.6{\cdot}10^{-3}$; 4) 1/10 and $4{\cdot}10^{-3}$; 5) 1/2 and $5.1{\cdot}10^{-3}$); b) $t = 310^3$ sec, $\xi_1 = 0$, $\xi_2 = 0$ (1) $\xi_0 = 1/10$, $h = 4{\cdot}10^{-3}$ m; 2) 1/5 and $3.6{\cdot}10^{-3}$; 3) 1.6 and $3.7{\cdot}10^{-3}$; 4) 2/11 and $3.6{\cdot}10^{-3}$; 5) 4/17 and $3.5{\cdot}10^{-3}$). u, °C.

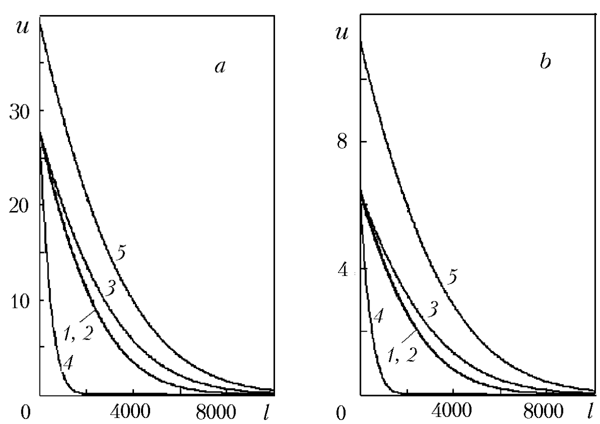

Fig. 3. Temperature distribution in the half-space depth at various flows and $t = 10^3$ sec [a) $\sigma_0 = 5{\cdot}10^8$ C/m^3, $p = 0$; b) $\sigma_0 = 0$, $p = 20$ J/(m^2·sec)]: 1) $\xi_0 = 0$, $\xi_1 = 0$, $\xi_2 = 0$, $h = 5{\cdot}10^{-3}$ m; 2) 1/4, 1/4, 1/4, and $5{\cdot}10^{-3}$; 3) 1/5, 1/5, 1/5, and $4{\cdot}10^{-3}$; 4) 1/10, 3/5, 3/5, and $2.3{\cdot}10^{-2}$; 5) 0, 1/4, 1/4, and $3.6{\cdot}10^{-3}$. u, °C.

Figure 2a gives the curves for various values of the time delay of magnetic polarization within the limits $0 \le \xi_2 \le 1/2$, $\xi_0 = 0$, $\xi_1 = 0$; Fig. 2b gives the curves for the delay times of conduction currents at $0 \le \xi_0 < 1/4$, $\xi_1 = 0$, $\xi_2 = 0$. With increasing time $t_0 = \xi_0 T$ the heating weakens.

Figure 3a presents the curves for various delay times in the presence on the surface of charges oscillating along the surface of Γ in time with the primary field: $\sigma_{\mathrm{Re}} = \mathrm{Re}\,(\sigma_0 \Phi \exp(-i\omega t)$. The presence of charges considerably intensifies the heating of the medium. Figure 3b presents the medium heating curves with heat outflow through the Γ plane.

NOTATION

A, B, complex strength amplitudes of the electric component of the primary TE- and TH-fields, respectively, V/m; a, F, G, P, Q, f, f_j, g_i, g, v_j, v, λ, ξ_m, Φ, auxiliary functions and constants; **B**, magnetic induction, kg/(sec^2·A); C, specific heat capacity of the medium in D_2, J/(K·kg); c, velocity of light in free space; **D**, electric induction, C/m^2; **E**, electric field strength, V/m; $\mathbf{E}_j$, complex electric field amplitudes, V/m; $\mathbf{e}_x$, bold$_y$, $\mathbf{e}_z$, Cartesian unit vectors; **H**, magnetic field strength, A/m; $\mathbf{H}_j$, complex magnetic field amplitudes, A/m; h, skin layer depth, m; I_s, amplitude of surface eddy currents, A/m; i, complex unit; **j**, surface current density, A/m; j_s, j_p, complex parameters of currents **j**, A; **J**, conduction current density, A/m^2; K_h, heat conductivity coefficient of the medium substance, kg·m/(sec^3·K); k, k_j, wave numbers of the field, 1/m; **m**, magnetic polarization, kg·/(sec^2·A); **n**, normal to the Γ surface; p, heat flow, J/(m^2·sec); **P**, electric polarization, C/m^2; **r**, radius vector, m; T, field period, sec; t, time, sec; t_0, t_1, t_2, delay times of the medium in the region of D_2, sec; u, temperature, ^{o}C; $\mathbf{W}^{(+1)}$, $\mathbf{W}^{(+2)}$, basic plane fields; x, y, z, Cartesian coordinates, m; x_j, y_j, complex stress amplitudes of the electric component of the TE- and TH-fields, respectively, V/m; α_j, complex propagation parameters of the primary field, 1/m; γ, specific electric conductance of the medium, 1/(Ω·m); ε_0, μ_0, electric and magnetic constants; ε_j, dielectric constant of the medium, Φ/m; θ_0, φ_0, angles of the spherical coordinate system defining the direction of propagation of the primary field energy, rad; κ, dielectric susceptibility of the medium; μ_j, magnetic permeability, H/m; ρ, density of the medium substance, kg/m^3; σ, complex surface density of charges, C/m^2; σ_0, complex amplitude for σ, C/m^2; χ, magnetic susceptibility of the medium; ω, circular frequency of the field, 1/m. Subscripts: Im, imaginary part; j, medium number: $j = 1$ and $j = 2$, media in half-spaces $z < 0$ and $z > 0$, respectively; m, delay time index, $m = 0, 1, 2$; p, potential; s, solenoidal; Re, real part; h, heat conductivity; 0, initial; ′ , reflected.

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